Calculus Warm Up # 7-2

BC - MC practice:

Pick up a half sheet on the stool.

$x_0 = 0$	$f(x_0) = 2$
$x_1 = 2$	$f(x_1) \approx 6$
$x_2 = 4$	$f(x_2) \approx 10$

Consider the differential equation $\dfrac{dy}{dx} = \dfrac{Ax^2 + 4}{y}$, where A is a constant.

Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 2$. Euler's method, starting at $x = 0$ with a step size of 2, is used to approximate $f(4)$. Steps from this approximation are shown in the table above. What is the value of A ?

(A) $\frac{1}{2}$

(B) 2

(C) 5

(D) $\frac{13}{2}$

Warm-up

$f(0) = 2 \quad f'(0) = \frac{4}{2} = 2$

$\Delta y = \Delta x\left(\frac{dy}{dx}\right) \qquad y + \Delta y$

$2 \cdot 2 = 4 \qquad 2 + 4 = 6 \rightarrow f(2) \approx 6$

given

could have started here:

$f(2) \approx 6 \quad f'(2) = \dfrac{A(4)+4}{6}$

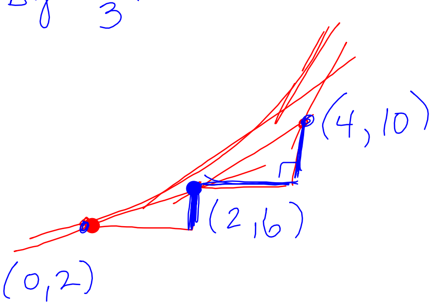
$= \frac{2}{3}(A+1) \qquad \Delta y = \frac{4}{3}(A+1)$

given: $f(4) \approx 10$

$y + \Delta y \approx 10$

$6 + \frac{4}{3}(A+1) \approx 10$

$\boxed{A = 2}$



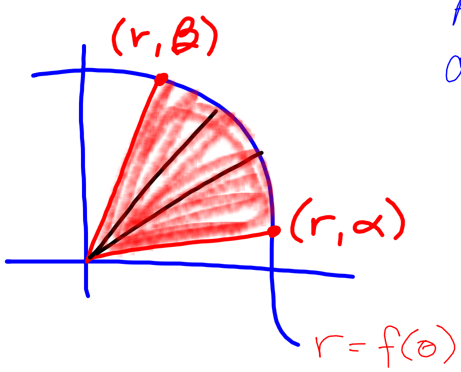
HW ?'s: (posted after tonight's hw slide)

7, 11,

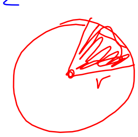
Due Friday

Classwork	HW Quiz by group
WU's (2 weeks)	10.8, p. 623 (2 days)
2009 FR	10.9, p. 630
	12.5, p. 715

12.5 Polar Area



Area of a sector $= \frac{1}{2}\theta r^2$



θ = radians

Polar function $r = f(\theta)$

continuous & non-negative

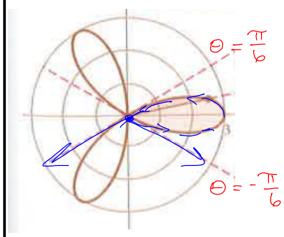
We will partition the region on [α,β] into n subintervals and sum the areas:

$$A = \lim_{n\to\infty} \sum \tfrac{1}{2}[f(\theta)]^2 \Delta\theta = \frac{1}{2}\int_{\alpha}^{\beta} [f(\theta)]^2 \, d\theta$$

Area of one petal on a rose curve.

r = 3 cos 3θ

$r = f(\theta) = 3\cos 3\theta$



$$A = \frac{1}{2}\int_{\alpha}^{\beta}[f(\theta)]^2\,d\theta$$

$$A = \frac{1}{2}\int_{-\frac{\pi}{6}}^{\pi/6}(3\cos 3\theta)^2\,d\theta$$

$$= \frac{1}{2}\int_{-\pi/6}^{\pi/6}(9\cos^2 3\theta)\,d\theta$$

$$= \frac{9}{2}\int_{-\pi/6}^{\pi/6}\frac{1+\cos 6\theta}{2}\,d\theta$$

$$= \frac{9}{4}\int_{-\frac{\pi}{6}}^{\pi/6}(1+\cos 6\theta)\,d\theta$$

$$= \frac{9}{4}\left[\theta + \frac{1}{6}\sin 6\theta\right]_{-\pi/6}^{\pi/6}$$

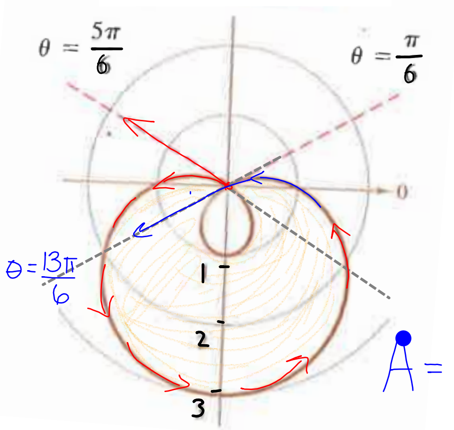
$$= \frac{9}{4}\left(\frac{\pi}{6}+\frac{\pi}{6}\right)$$

$$\boxed{= \frac{3\pi}{4}}$$

Area of the region between the inner and outer loops:

$$A = A_o - A_i$$

$r = 1 - 2\sin\theta$



Intervals:

Inner Loop: $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$

Outer Loop: $\left[\frac{5\pi}{6}, \frac{13\pi}{6}\right]$

$$A = \frac{1}{2}\left[\int_{5\pi/6}^{13\pi/6}(1-2\sin\theta)^2\,d\theta - \int_{\frac{\pi}{6}}^{5\pi/6}(1-2\sin\theta)^2\,d\theta\right]$$

the integrand... $1 - 4\sin\theta + 4\sin^2\theta$

$1 - 4\sin\theta + 4\left(\frac{1-\cos 2\theta}{2}\right)$

$3 - 4\sin\theta - 2\cos 2\theta$

now integrate:

$$A = \frac{1}{2}\left[\left(3\theta + 4\cos\theta - \sin 2\theta\right)\Big|_{\frac{5\pi}{6}}^{\frac{13\pi}{6}} - \left(3\theta + 4\cos\theta - \sin 2\theta\right)\Big|_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\right]$$

$$= \left(2\pi + \frac{3\sqrt{3}}{2}\right) - \left(\pi - \frac{3\sqrt{3}}{2}\right)$$

$$= \pi + 3\sqrt{3}$$

HW:

p. 715, # 3 - 11 odd

The given intervals of integration are for half the region. They have symmetry there so area will be double the integral:

3. $[0, \frac{\pi}{6}]$ 5. $[0, \frac{\pi}{4}]$ 7. $[-\frac{\pi}{2}, \frac{\pi}{2}]$ 9. $[\frac{2\pi}{3}, \pi]$

11. inner loop: $[0, \frac{2\pi}{3}]$

outer loop in exercise #9

HW Questions: p. 630

7) $f(x) = \frac{1}{2x-5}$; $c = -3$

$$\frac{1}{2(x+3)-5-6} = \frac{1}{-11\left(1-\frac{2}{11}(x+3)\right)} = \frac{-\frac{1}{11}}{1-\frac{2}{11}(x+3)}$$

$$a = -\frac{1}{11}, \quad r = \frac{2}{11}(x+3)$$

$$\sum_{n=0}^{\infty}\left(-\frac{1}{11}\right)\left(\frac{2}{11}(x+3)\right)^n$$

$$\left|\frac{2(x+3)}{11}\right| < 1$$

$$-11 < 2(x+3) < 11$$

$$-\frac{11}{2} < x+3 < \frac{11}{2}$$

$$\boxed{-\frac{17}{2} < x < \frac{5}{2}}$$

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11) $$\frac{3x}{(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{x-1}$$

$$= \frac{2}{x+2} + \frac{1}{x-1}$$

$3x = A(x-1) + B(x+2)$

$x = 1 \rightarrow 3 = B(3)$, $B = 1$

$x = -2$: $-6 = A(-3)$, $A = 2$

$$\frac{2}{2\left(1+\frac{x}{2}\right)} + \frac{1}{-(1-x)}$$

$$\frac{1}{1-\left(-\frac{x}{2}\right)} + \frac{-1}{1-x}$$

$a = 1$, $r = -\frac{x}{2}$

$a = -1$, $r = x$

$$f(x) = \sum_{n=0}^{\infty}\left[\left(-\frac{x}{2}\right)^n - x^n\right]$$

$$= \sum_{n=0}^{\infty} x^n\left[\left(-\frac{1}{2}\right)^n - 1\right]$$

$$|x| < 1$$

$$\boxed{-1 < x < 1}$$

2009 FR #4-6

4) a) $\frac{4}{3}$

b) $\frac{8}{15}$

c) $V = \pi\int_0^4 \left[\left(2-\frac{x}{2}\right)^2 - \left(2-\sqrt{x}\right)^2\right]dx$

5a) $y - e^2 = -4e^2(x-1)$

b) g has a max @ $x = -1$

c) $g''(-1)$ is negative

d) $2e^2$

6a) $a(36) \approx \frac{11}{8}\ m/s^2$

b) -75 meters

c) $8 < t < 20$

$32 < t < 40$

d) $s(8) = s(0) + \int_0^8 v(t)\,dt$

$s(8) \approx 7 + \left(\text{trap approx}\right)$

underestimate b/c $a(t) = v'(t) > 0$

$s(8) \approx 7 + 32 > 30$